Topological defects and the evasion of Derrick's theorem in curved spacetime

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Topological Defects

What are they?

They are stable configurations of matter, formed at the phase transitions in the very early universe.

- **Discrete** symmetry → Domain Walls
- **Cylindrical** symmetry → Cosmic Strings
- **Spherical** symmetry → Monopoles
- **Complicated** symmetry groups → Textures
- Other forms

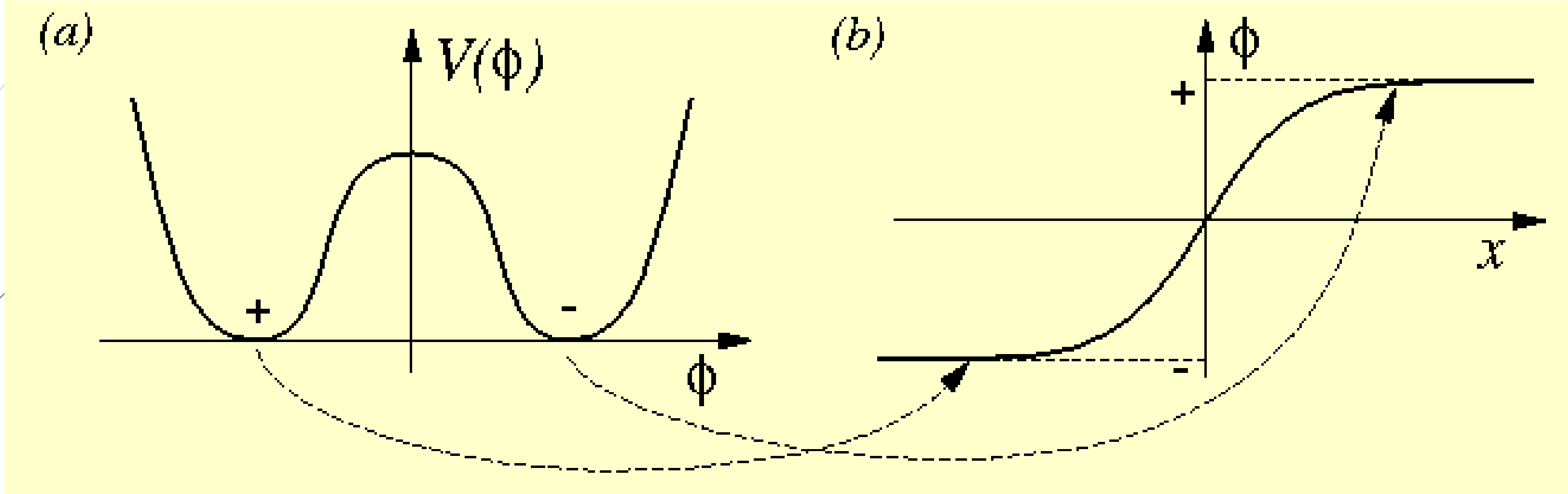

Fig.1 Domain walls are associated with models in which there is more than one separated minimum

Derrick's Theorem

- In 3 or more dimensions any finite energy initially static field solutions to a nonlinear Klein-Gordon equation are unstable and energetically favored to shrink and collapse.
- **Can it be evaded?**
- **What field configuration survives in a curved background?**

Evasion of Derrick's theorem

- Consider the static, spherically symmetric Grumiller metric **[1]**,

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

$$f(r) = 1 - \frac{2Gm}{r} + 2\,b\,r - \frac{\Lambda}{3}r^2 \quad (2)$$

where **Λ** is the cosmological constant and **b** is the Rindler acceleration parameter. The energy functional has the form,

$$\begin{aligned} E &= \int d^3x\sqrt{-g}\,T_0^0 \\ &= 4\pi\int_{r_1}^{r_2}\left[\frac{1}{2}f(r)\left(\frac{d\Phi}{dr}\right)^2 + V(\Phi)\right]r^2dr \end{aligned} \quad (3)$$

- Derrick's theorem + flat space → **Instability**
- Derrick's theorem + curved space → **??**

We rescale the field using a parameter **a** getting,

$$E_\alpha = 4\pi \int_{r_1}^{r_2} \left[r^2 f(r) \left(\frac{d\Phi_\alpha}{dr} \right)^2 + V(\Phi_\alpha) r^2 \right] dr$$

where,

$$\Phi_\alpha \equiv \Phi(\alpha r)$$

- for the existence of a static solution a necessary condition is **[2]**,

$$\frac{1}{4\pi}\frac{dE}{d\alpha}\bigg|_{\alpha=1} = I_1 + I_2 + I_3 = 0$$

where,

$$I_1 = -\int_{r_1}^{r_2} r^3 f'(r)\left(\frac{d\Phi}{dr}\right)^2 dr$$

$$I_2 = -\int_{r_1}^{r_2} r^2 f(r)\left(\frac{d\Phi}{dr}\right)^2 dr$$

$$I_3 = -\int_{r_1}^{r_2} r^2\, V(\Phi)\, dr$$

- Therefore we need that,

$$I_1 > 0 \;\rightarrow\; f'(r) < 0$$

- Flat space → Derrick's theorem is applicable
- Schwarzchild metric → Derrick's theorem is applicable
- Reissner-Nordstrom → Derrick's theorem is applicable
- Grumiller metric → **??**

- For a symmetry breaking potential,

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - \eta^2\right)^2 \qquad (4)$$

and an effective potential that is approximately,

$$\frac{E(r_0)}{4\pi\lambda^{1/2}\eta} \simeq 4\,\bar{r}_0^2\, f(\bar{r}_0) + \bar{V}(0)\,\bar{r}_0^2 \qquad (5)$$

using (**thin wall approximation**),

$$\Delta r \simeq \lambda^{-1/2}\eta^{-1}$$

$$\Delta\Phi = 2\eta$$

$$\bar{r}_0 \equiv \lambda^{1/2}\,\eta\,r_0$$

$$\bar{V}(0) \equiv V(0)/(\lambda\eta^4)$$

Conditions for Stability

Condition I:

- The effective potential should have at least one local **minimum**.

Condition II:

- The metric function f(r) should be **positive** at that local minimum so that it is **not** hidden by a horizon and no negative energy (ghost) instabilities appear.

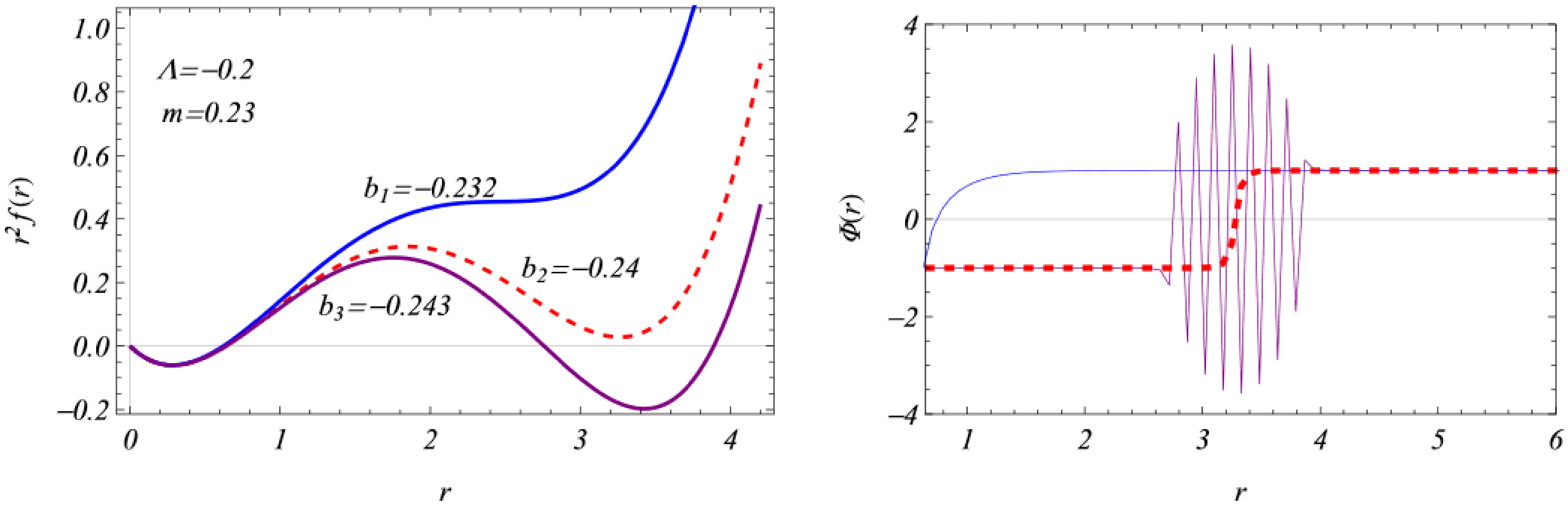

Fig 2. The three different behaviors of the metric function and their corresponding field configuration. Only the red, dotted lines correspond to metastable solutions.

- The existence of a metastable solution depends on the range of the metric parameters **Λ** and **b** (for fixed **m**) which can **only** be found numerically by minimizing the energy functional (3).

- This range is a subspace of the range that satisfies the conditions I, II

$$|\Lambda| \in \left[\left|\frac{27b\,Gm - 2\sqrt{(9b\,Gm+1)^3}+2}{18G^2m^2}\right|, \left|-\frac{(12b\,Gm+1)^{3/2}-18b\,Gm-1}{18G^2m^2}\right|\right] \qquad (6)$$

We call (6) the **candidate range**.

- Using numerical analysis we obtained the precise subspace of the candidate range that corresponds to metastable field configuration solutions.

m = 0

```
Clear[ll, b, imax, f];
Do[abmin = -Sqrt[8 Abs[ll] / 27];
 abmax = -Sqrt[9 Abs[ll] / 27];
 Do[Clear[imax, f];
  imax = 200;
  roots = NSolve [r^2 + 2 b r^3 - ll r^4 / 3 == 0, r];
  r1 = roots[[2, 1, 2]];
  rmin = r1;
  rmax = 12;
  dx = (rmax - rmin) / imax;
  r[i_] := i dx + rmin;
  fp[i_] := (f[i] - f[i - 1]) / dx;
  e[i_] := (r[i]^2 + 2 b r[i]^3 - (ll / 3) r[i]^4) fp[i]^2 + r[i]^2 (f[i]^2 - 1)^2 / 2;
  f[0] = -1;
  f[imax] = 1;
  etot = dx Sum[e[i], {i, 1, imax}];
  r0 = 10;
  ft[x_] := Tanh[5 (x - r0)];
  ftab = Table[{f[i], ft[r[i]]}, {i, 1, imax - 1}];
  sol1 = FindMinimum[etot, ftab, MaxIterations → 1000000, AccuracyGoal → 6];
  limit1 = f[IntegerPart[0.8 * imax / rmax]] /. sol1[[2]];
  If[limit1 <= -0.9 && limit1 >= -1, Print["{", ll, ",", b "}" ","]
   Break[]], {b, abmin, abmax, -0.0001}], {ll, -0, -0.6, -0.01}]
```

candidate range

minimization of energy functional

check for stability

It’s easy to see that an analytic fit of the form,

$$\Phi(r) = Tanh\left(q(r - r_0)\right)$$

provides a great ft to the numerically obtained solution.

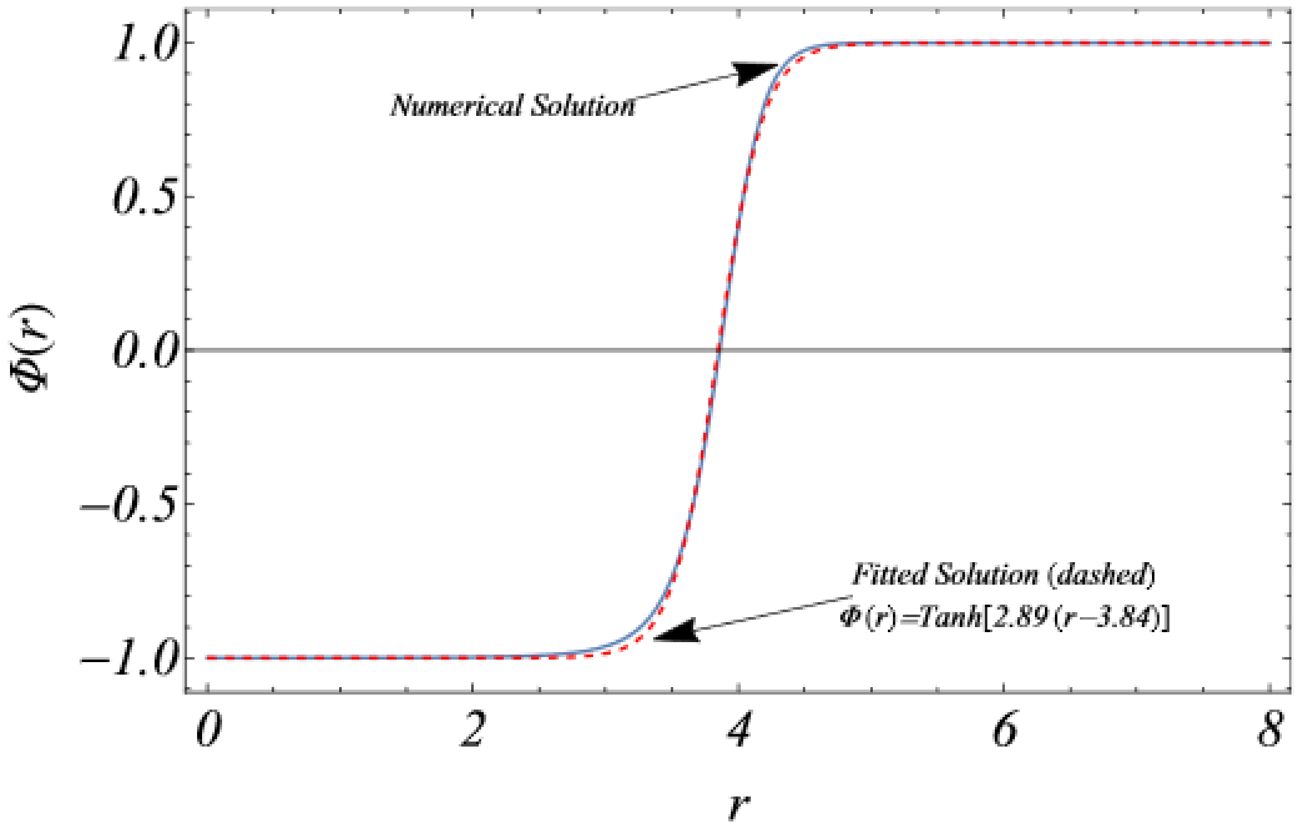

Fig3. We can see that for the values **b = -0.21**, **Λ = -0.14** and **m = 0** the analytic fits the numerical one excellently.

We have also considered the effects of the backreaction of the domain wall solution on the metric

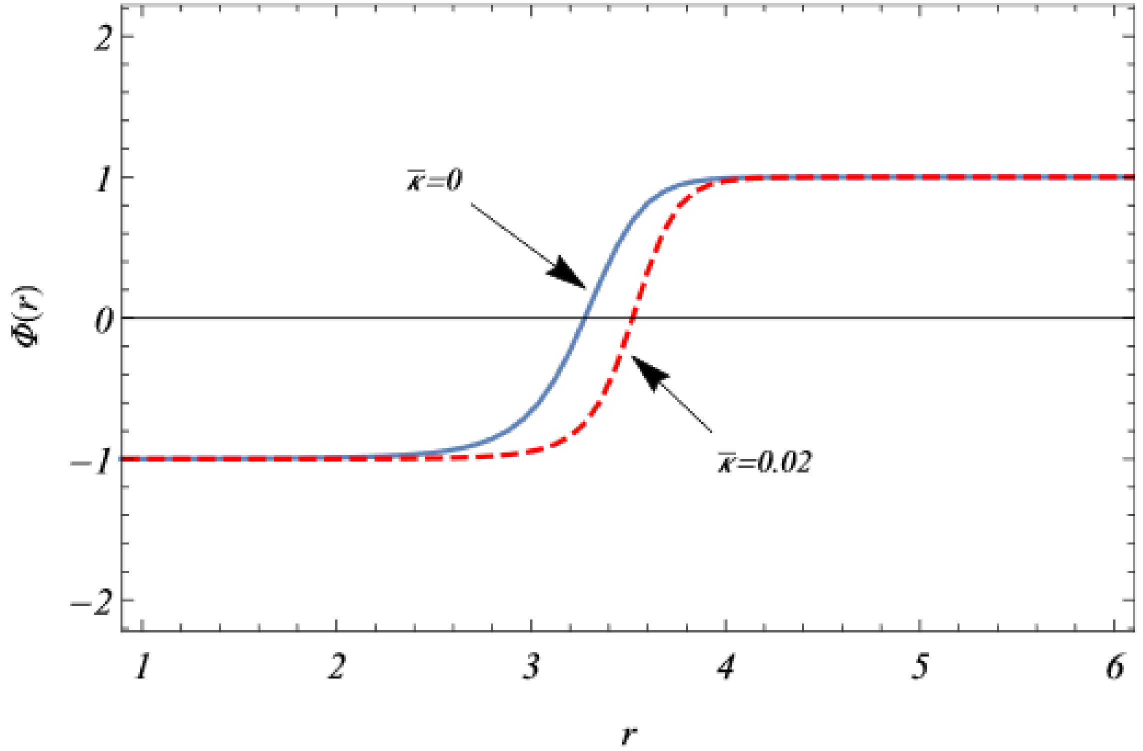

Fig.4 The backreaction changes not only the depth but also the position of minimum of the energy functional.

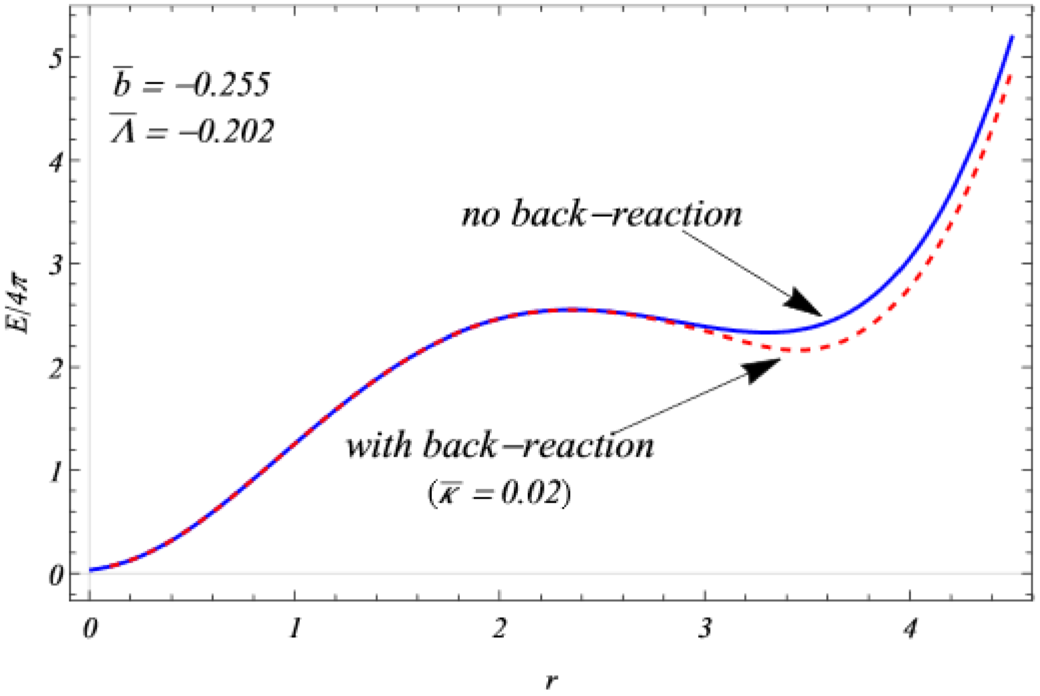

Fig.5 The effects of small backreaction on the static solution and on the energy functional.

Stability Regions (m=0)

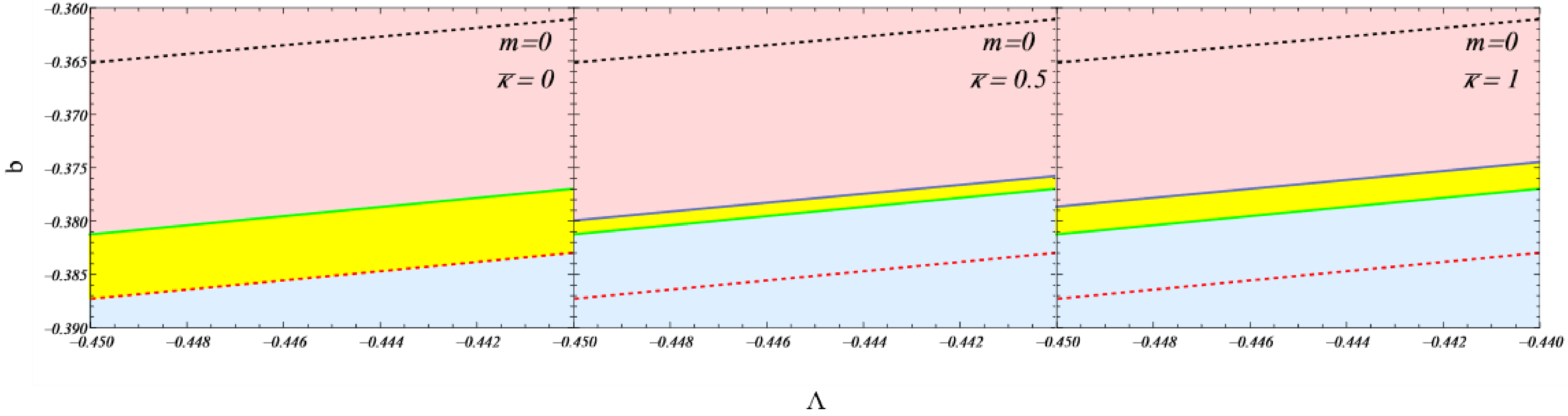

Fig.6 The yellow area corresponds to the stability area in the parameter space **b**, **Λ** for **m = 0**. We can see that the backreaction **k** tends to lower the energy minimum, thus the yellow area decreases from below.

Stability Regions (m=0.1)

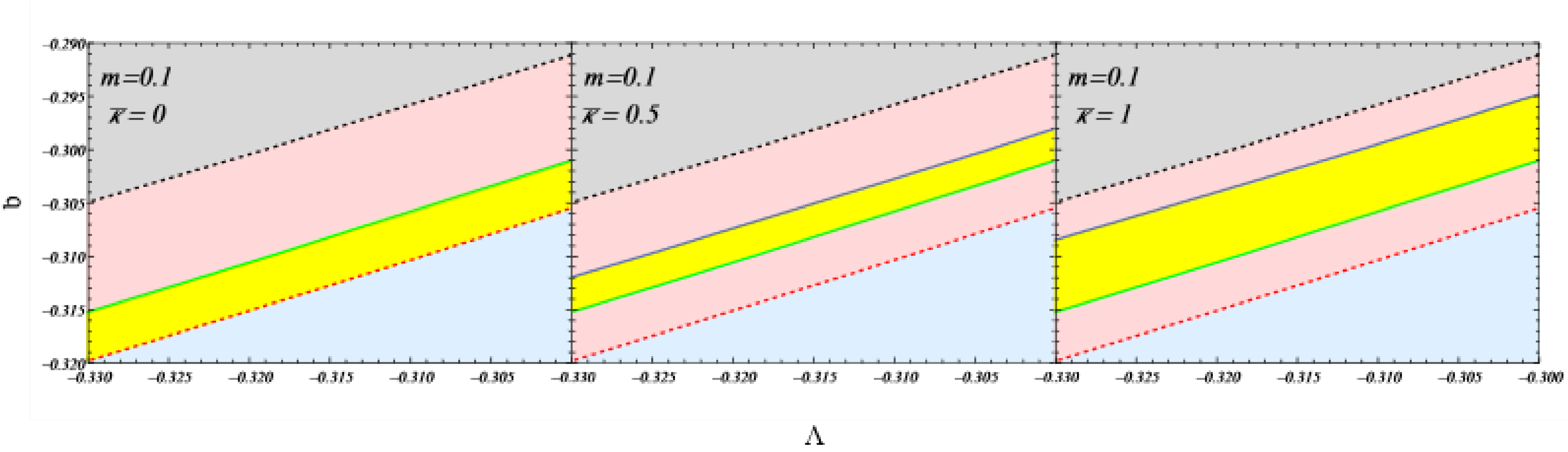

Fig.7 Same as fig.6 but for **m = 0.1**

Field Evolution

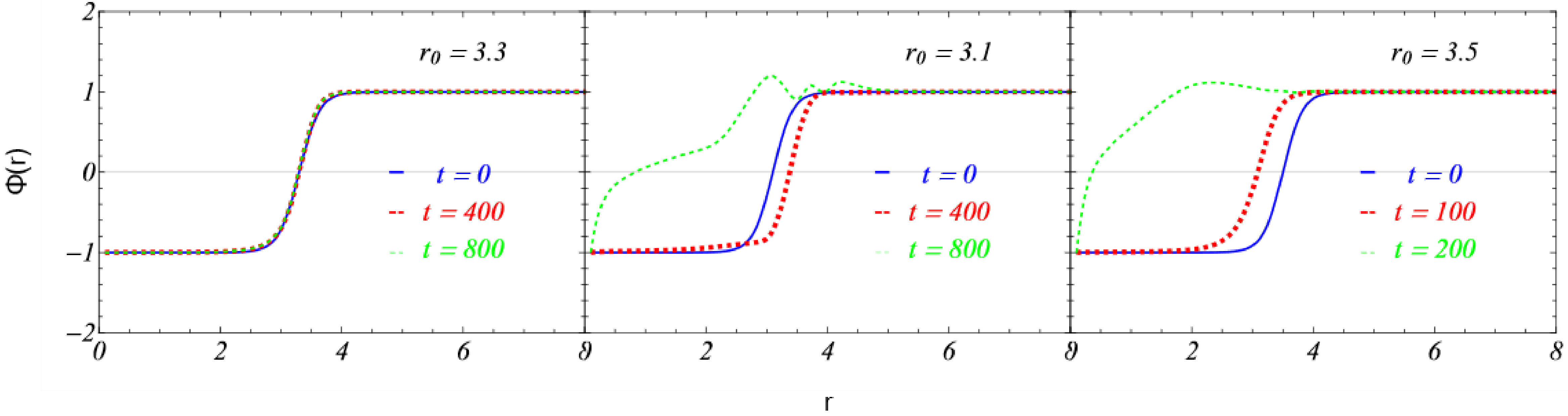

Fig.8 Simulation of the field configuration with parameter values **m = 0**, **b= - 0.25**, **Λ = -0.2** and initial wall approximated by the analytic fit with initial radius equal to the static solution (3.3), slightly smaller (3.1) and larger (3.5). The initial configuration remains static in the left panel but collapses in both other.

- Derrick’s theorem **can be evaded** in a curved non-trivial background!!
- These metastable spherical domain walls could produce very interesting observational signatures on a cosmological scale, such as characteristic lensing **[2-4]** and glitches in the galactic rotation curves.
- This study has been published in Physical Review D.

References

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Thank you for listening!!