Stable Spherical Fluid Shells in a Schwarzschild-Rindler-anti-de Sitter Metric

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Based on arXiv:2005.11702 by G. Alestas, G. V. Kraniotis and L. Perivolaropoulos, currently under peer-review

September 27, 2020

Overview

SRAdS Metric

- This spherically symmetric metric was first proposed by Grumiller and has the form:

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\begin{aligned}
ds^2 &= f(r)dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \\
f(r) &= 1 - \frac{2Gm}{r} + 2br - \frac{\Lambda}{3}r^2
\end{aligned}
\tag{1}
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where Λ is the cosmological constant and b is the Rindler acceleration term. The Rindler term balances the mass and the anti de Sitter terms allowing for stability.

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- It has been constrained by solar system observations such that $|b| < 3nm/sec^2$, it can lead to flat rotation curves and explains the Pioneer anomaly.
- It emerges generically as a vacuum solution of spherically symmetric scalar-tensor theories, as well as a vacuum solution in the conformal Wey gravity.

D. Grumiller, Phys.Rev.Lett 105 211303 (2010), S. Carloni et al, Phys.Rev.D 83 124024 (2011), L. Iorio, Mon.Not.Roy.Astron.Soc. 419 2226-2232 (2012), P. D. Manheim et al, Astrophys.J. 342 635-638 (1989)

Thin Spherical Shells in GR

- They are $2 + 1$ boundary hypersurfaces with energy momentum tensor

$$S_j^i \equiv \int_{R^-}^{R^+} T_j^i \, dr = \mathrm{diag}(-\sigma, p, p) \tag{2}$$

  where R is the is the shell radius, r is the radial coordinate of the $3 + 1$ dimensional spacetime, σ is the surface energy density and p is the surface pressure on the shell hypersurface with equation of state $p = p(\sigma)$.

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- The thin shell interpolates between an interior and an exterior spherically symmetric metric. The exterior metric is related to the interior metric in the context of the Israel junction conditions.

W. Israel, Nuovo Cim. B 44S10 1 (1966)
N. Sen, Annalen der Physik 378 365-396 (1924)

Thin Spherical Shells as a Black Hole Alternative (1/2)

Enter Gravastars!

- Gravastars were originally proposed by Mazur and Mottola as a new final state of the gravitational collapse of stars, a black hole alternative. Their model consisted of a thin shell that separated spacetime into two pieces. The interior region governed by the de Sitter metric and an exterior region described by a Schwarzschild backgound.

Thin Spherical Shells as a Black Hole Alternative(2/2)

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- Externally, a gravastar appears similar to a black hole. It is visible by the high-energy radiation it emits while consuming matter, and by the Hawking radiation it creates. Therefore it would produce an identical observational signature that extends to lensing if the shell isn't transparent to radiation.
- Turns out they are not so stable when they rotate rapidly.

P. O. Mazzur et al, Proc.Nat.Acad.Sci. 101 9545-9550 (2004)
M. Visser et al, Class.Quant.Grav. 21 1135-1152 (2004)
V. Cardoso et al, Phys.Rev. D 77.124044 (2007)

Motivation

It has been shown that this metric supports the existence and metastability of spherical topological defects in the form of domain walls, which arise as solutions to dynamical scalar field equations.

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Can this be extended and generalized to fluid thin shells using the Israel formalism?

G. Alestas et al, Phys.Rev. D 99 064026 (2019)

Questions to address

- Are there static, stable fluid shell solutions in a SRAdS background geometry?

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- If yes, what are the conditions for their stability given the equation of state of the fluid shell?
- What is the metric parameter range for shell stability and how does the stability radius change as a function of these parameters?

Existence and Stability Analysis (1/4)

- Consider a thin shell with radius R interpolating between an interior (-) and an exterior ($+$) SRAdS metric,

$$ds^2 = f_\pm(r_\pm)dt^2 - \frac{dr_\pm^2}{f_\pm(r_\pm)} - r_\pm^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

where,

$$f_\pm(r_\pm) = 1 - \frac{2m_\pm(r_\pm)}{r_\pm} \tag{4}$$

and

$$m_\pm(r_\pm) = m_\pm - br_\pm^2 + \frac{\Lambda}{6}r_\pm^3. \tag{5}$$

Existence and Stability Analysis (2/4)

We impose two types of conditions:

- The continuity of the metric on the shell ($r_- = r_+ = R$), which leads to

$$\begin{aligned} t_- &= \frac{f_+(R)}{f_-(R)} t_+ \\ \frac{dr_-}{dr_+} &= \frac{f_-(R)}{f_+(R)}. \end{aligned} \tag{6}$$

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- And the Israel junction conditions expressed through a discontinuity of the extrinsic curvature on the shell hypersurface Σ. For the extrinsic curvature tensor of our metric,

$$K_{ij} = \sqrt{f_\pm(r_\pm)} \mathrm{diag}\left(\frac{\frac{1}{2} f'_\pm(r_\pm)}{f_\pm(r_\pm)}, \frac{1}{r_\pm}, \frac{1}{r_\pm} \right) \tag{7}$$

Existence and Stability Analysis (3/4)

the Israel conditions take the form,

$$\begin{aligned} \sigma &= -\frac{1}{4\pi R}\left[\left[\sqrt{1-2m_\pm(R)/R+\dot{R}^2}\right]\right] \\ p &= \frac{1}{8\pi R}\left[\left[\frac{1-m_\pm(R)/R-m_\pm(R)'+\dot{R}^2+R\ddot{R}}{\sqrt{1-2m_\pm(R)/R+\dot{R}^2}}\right]\right] \end{aligned} \tag{8}$$

for a dynamic shell.

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for a dynamic shell.

- Therefore the energy conservation equation on the shell is,

$$\frac{d}{d\tau}(\sigma R^2)+p\frac{d}{d\tau}R^2=0 \tag{9}$$

identical to that of a particle moving in one dimension with zero energy.

Existence and Stability Analysis (4/4)

- The potential of the shell for the case of the SRAdS metric is,

$$V(R) = 1 - \frac{m_- + m_+}{R} + 2bR - \frac{\Lambda R^2}{3} - \frac{(m_- - m_+)^2}{16\pi^2 R^4 \sigma(R)^2} - 4\pi^2 \sigma(R)^2 R^2 \tag{10}$$

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- And the conditions for the existence of a static, stable shell are,

$$\begin{aligned} V(R) &= 0 \\ V^{'}(R) &= 0 \\ V^{''}(R) &> 0 \end{aligned} \tag{11}$$

Case I: Vacuum Fluid Shell (1/5)

- It is the simplest case of a stable spherical shell. Similar to the case of a stable domain wall which has already been shown.

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- It is the simplest case of a stable spherical shell. Similar to the case of a stable domain wall which has already been shown.
- The energy conservation equation gives us,

$$\sigma(R) = \sigma_0 = const. \tag{12}$$

- Therefore, the system conditions for existence and stability take the form,

$$\begin{aligned} V(R) &= 1 - \frac{m_- + m_+}{R} + 2bR - \frac{\Lambda R^2}{3} - \frac{(m_- - m_+)^2}{16\pi^2 R^4 \sigma_0^2} - 4\pi^2\sigma_0^2 R^2 = 0 \\ \frac{\partial V}{\partial r}\Big|_{r=R} &= 2b + \frac{m_- + m_+}{R^2} - \frac{2\Lambda R}{3} + \frac{(m_- - m_+)^2}{4\pi^2 R^5 \sigma_0^2} - 8\pi^2\sigma_0^2 R = 0 \\ \frac{\partial^2 V}{\partial r^2}\Big|_{r=R} &= -\frac{2\Lambda}{3} - \frac{2(m_- + m_+)}{R^3} - \frac{5(m_- - m_+)^2}{4\pi^2 R^6 \sigma_0^2} - 8\pi^2\sigma_0^2 > 0. \end{aligned} \tag{13}$$

Case I: Vacuum Fluid Shell (2/5)

- The solution of this system is,

$$\Lambda(R,\sigma_0) = \frac{15(m_- - m_+)^2}{16\pi^2 R^6 \sigma_0^2} + \frac{6(m_- + m_+) - 3R}{R^3} - 12\pi^2\sigma_0^2 \tag{14}$$

$$b(R,\sigma_0) = \frac{3(m_- - m_+)^2 + 8\pi^2[3(m_- + m_+) - 2R]R^3\sigma_0^2}{16\pi^2\sigma_0^2 R^5} \tag{15}$$

$$R > 3(m_- + m_+) \equiv R_{min} \tag{16}$$

$$\sigma_0 \equiv \frac{\sqrt{15}\sqrt{-\frac{(m_- - m_+)^2}{R^3(3m_- + 3m_+ - R)}}}{4\pi} + \Delta\sigma > \frac{\sqrt{15}\sqrt{-\frac{(m_- - m_+)^2}{R^3(3m_- + 3m_+ - R)}}}{4\pi} \equiv \sigma_{0min} \tag{17}$$

where $\Delta\sigma$ allows for small perturbations on the surface density, higher than that of its minimum value σ_{0min}.

Case I: Vacuum Fluid Shell (3/5)

Due to the lower limits of R and σ we derive the boundaries of the stability (Λ, b) parameter space:

$$b_{min} \to -\frac{1}{6(m_+ + m_-)}, \Lambda_{max} \to -12\pi^2\sigma_0^2 \tag{18}$$

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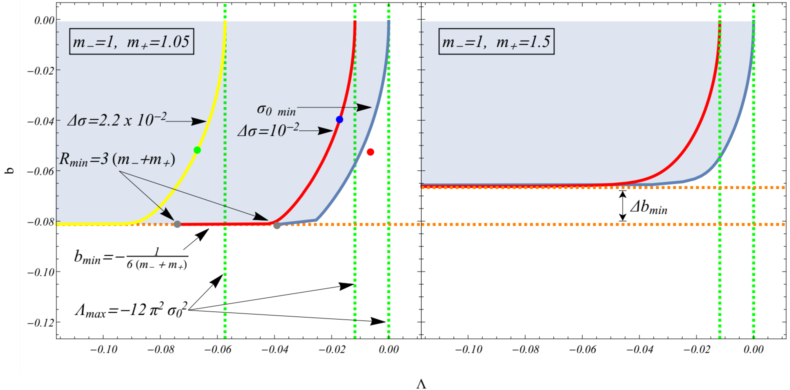
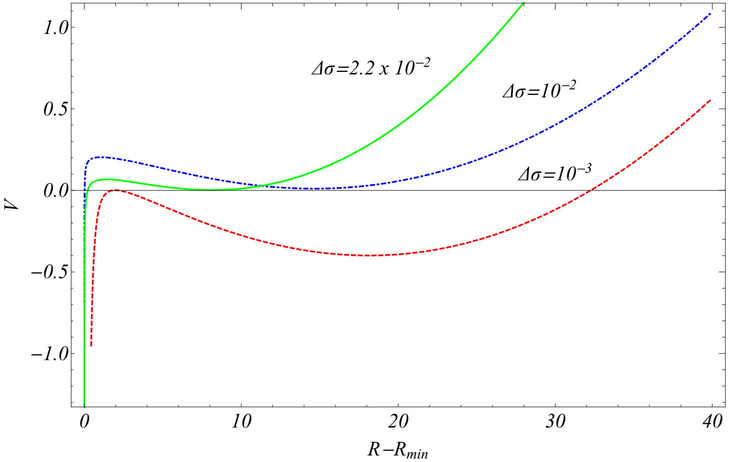
These boundaries are demonstrated for $(p = -\sigma)$ in the figure below,



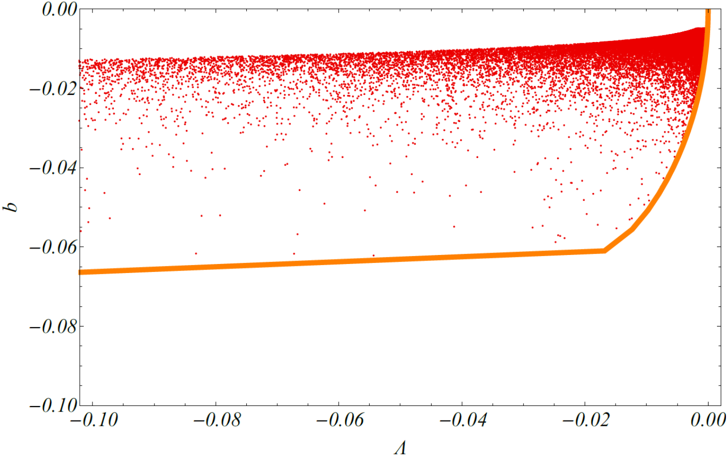
Case I: Vacuum Fluid Shell (4/5)

If we plot the potential for the parameter values that correspond to each of the colored points of the previous plot we see that the only viable solutions exist inside the stability region. This is shown in the figure below,



Case I: Vacuum Fluid Shell (5/5)

These results are also verified by performing a Monte-Carlo computational process for $N = 5 \times 10^4$ random points as shown below,



Case II: Stiff Matter Fluid Shell (1/3)

- The equation of conservation of energy takes the form,

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- And the system conditions are,

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Case II: Stiff Matter Fluid Shell (2/3)

- With solutions,

$$\Lambda(R,\sigma_0) = -\frac{9(m_- - m_+)^2 R^2}{16\pi^2\sigma_0^2} + \frac{6(m_+ + m_-) - 3R}{R^3} + \frac{84\pi^2\sigma_0^2}{R^8} \tag{21}$$

$$b(R,\sigma_0) = -\frac{(m_- - m_+)^2 R^3}{16\pi^2\sigma_0^2} + \frac{3(m_- + m_+) - 2R}{2R^2} + \frac{16\pi^2\sigma_0^2}{R^7} \tag{22}$$

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- And constraints,

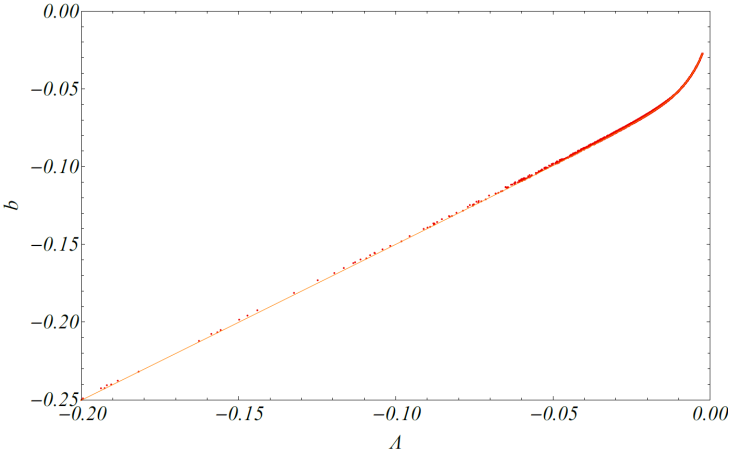
$$-4\pi\sqrt{\sigma_0^2(R^6 - 6m_+R^5 - 100\pi^2\sigma_0^2)} < \frac{3[(m_- - m_+)R^5 + 8\pi^2\sigma_0^2]}{\sqrt{3}} \quad (23)$$

$$\frac{3[(m_- - m_+)R^5 + 8\pi^2\sigma_0^2]}{\sqrt{3}} < 4\pi\sqrt{\sigma_0^2(R^6 - 6m_+R^5 - 100\pi^2\sigma_0^2)} \quad (24)$$

$$R^6 > 6m_+R^5 + 100\pi^2\sigma_0^2 \quad (25)$$

Case II: Stiff Matter Fluid Shell (3/3)

Performing the same Monte-Carlo computational process yields the following narrow stability region,



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- Lastly, for a pressureless dust fluid shell the conservation of energy gives us,

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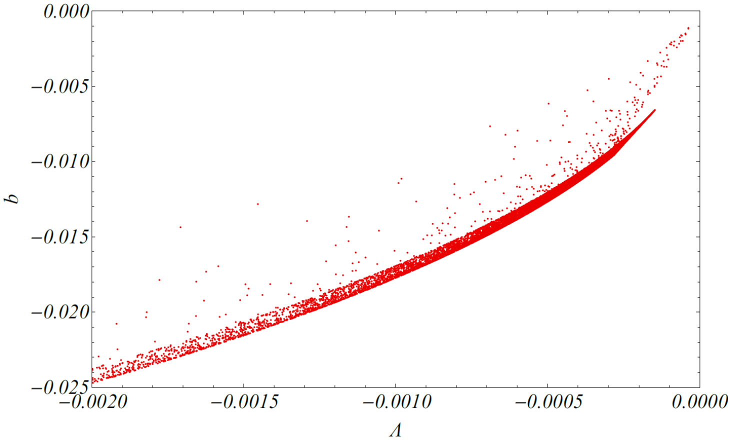
- With solutions to the system conditions for existence and stability,

$$\Lambda(R, \sigma_0) = \frac{3(m_- - m_+)^2}{16\pi^2 R^2 \sigma_0^2} + \frac{6(m_- + m_+) - 3R}{R^3} + \frac{36\pi^2 \sigma_0^2}{R^4} \tag{27}$$

$$b(R, \sigma_0) = \frac{(m_- - m_+)^2}{16\pi^2 R \sigma_0^2} + \frac{3(m_- + m_+) - 2R}{2R^2} + \frac{8\pi^2 \sigma_0^2}{R^3} \tag{28}$$

Case II: Dust Fluid Shell (2/2)

The Monte-Carlo map of the dust matter shell stability region with $m_{-} = 1$ and $m_{+} = 1.5$,



Summary

- We have demonstrated the existence of static, stable spherically symmetric thin fluid shells in a Schwarzschild-Rindler-anti-de Sitter (SRAdS) metric by finding the analytic conditions for stability and the corresponding range of values of metric parameters that admit stable fluid shells for different forms of fluid equation of state.

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- These shells have similarities with the well known gravastar shell structures.
- In our shell structures the interior de Sitter term of the gravastars is replaced by a combination of Rindler-anti-de Sitter terms present in a continuous form (same values both in the interior and in the exterior of the shell).

Looking Ahead

- The investigation of observational effects of such shell structures. For example signatures of such SRAdS shell structures in typical lensing patterns could be identified and compared to observed lensing patterns around black holes.

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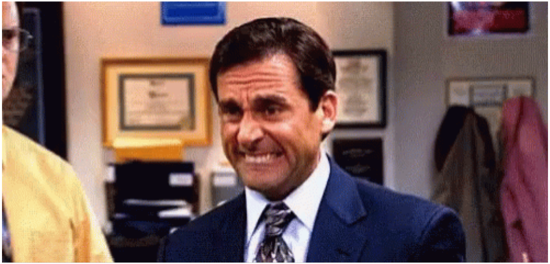
- The investigation of non-spherical junctions and shells. An interesting problem would be the study of joining rotating spacetimes in the presence of the cosmological constant.
- The investigation of alternative forms of metrics that may admit stable shell solutions.
- And the investigation of the dynamical evolution of the shell in the context of spherical symmetry and beyond.

Thank you for listening!!

The prophet who predicted the world would end in 2012, realizing that he forgot the calculate the uncertainty of ±8.



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Guest Stars!